EDDY CURRENTS IN CORES WITH TRIANGULAR CROSS-SECTIONS

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Synopsis

Numerical methods are recommended for the determination of the distribution of eddy current density in general triangular core-sections. Analytical expressions are obtained for components of magnetic field intensity and eddy current density in cores having cross-sections in the shape of a right-angled isosceles triangle.

α Introduction

Analytical solutions of eddy current equations are available for cores of rectangular cross-sections *{Mukerji, 2008}*. For cores with right-angled isosceles triangular cross-sections, analytical solutions for Laplace equation and wave equation are available *{Mukerji, 2008}*[1]. The objective of this paper is to present analytical solution for eddy current equation giving field distribution in cores with right-angled isosceles triangular cross-sections. Consider a long conducting core with a uniformly distributed winding on its surfaces that carries an alternating current with angular frequency ω. Let the winding currents be simulated by a current sheet on core surfaces with surface current density $K_o.e^{j\omega t}$, where $|K_o|$, indicates ampere–turns per unit core length. If the displacement currents are neglected, magnetic field outside the core winding will be zero. The eddy current equation for this two-dimensional problem found from Maxwell's equations is as follows:

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = j\omega\mu\sigma H_z \stackrel{\text{def}}{=} j\frac{2}{d^2}H_z \tag{1}$$

where, $\mu =$ permeability of the core and
$\sigma =$ conductivity of the core

Having solved eq. (1), the expression for the eddy current density in the core is found from Maxwell's equation, with displacement currents neglected

$$J = \nabla \times H \tag{2}$$

β Magnetic field distribution

The distribution of magnetic field in cores with cross-section in the shape of an arbitrary triangle can best be obtained by numerical methods *{Guru, 1998; Kraus, 1999}*. However, in an exceptional case as shown below, it is possible to obtain this distribution using analytical methods.

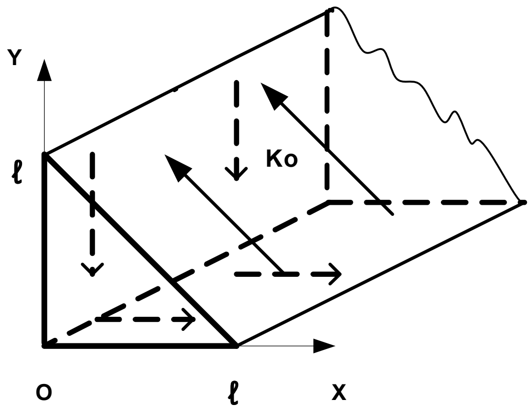

Figure 1 A core with triangular cross-section.

For the triangular section shown in Figure 1 the boundary conditions are as indicated below:

$$H_z|_{x=0} = K_0 . e^{j\omega t} \quad \text{over } 0 < y < \ell \tag{3.1}$$

$$H_z|_{y=0} = K_0 . e^{j\omega t} \quad \text{over } 0 < x < \ell \tag{3.2}$$

$$H_z|_{y=\ell - x} = K_0 . e^{j\omega t} \quad \text{over } 0 < x < \ell \tag{3.3}$$

Let us split up the field H_z into two partial field components, each satisfying eddy current equation:

$$H_z = H_{1z} + H_{2z} \tag{4}$$

Further, let the first field component be subjected to the following boundary conditions:

$$H_{1z}|_{x=0} = K_0 . e^{j\omega t} \quad \text{over } 0 < y < \ell \tag{5.1}$$

$$H_{1z}|_{x=\ell} = -K_0 . e^{j\omega t} \quad \text{over } 0 < y < \ell \tag{5.2}$$

$$H_{1z}|_{y=0} = K_0 . e^{j\omega t} \quad \text{over } 0 < x < \ell \tag{5.3}$$

$$H_{1z}|_{y=\ell} = -K_0 . e^{j\omega t} \quad \text{over } 0 < x < \ell \tag{5.4}$$

Solution of eddy current equation for H_{1z}, in view these boundary conditions are as follows:

$$H_{1z} = -\sum_{m-odd}^{\infty} K_0 . \frac{4}{m\pi} . \left[\sin\left(\frac{m\pi}{\ell} . y\right) . \frac{\sinh \alpha_m .(x - \ell/2)}{\sinh(\alpha_m . \ell/2)} + \sin\left(\frac{m\pi}{\ell} . x\right) . \frac{\sinh \alpha_m .(y - \ell/2)}{\sinh(\alpha_m . \ell/2)} \right] . e^{j\omega t} \tag{6}$$

Where,

$$\alpha_m = \sqrt{\left(\frac{m\pi}{\ell}\right)^2 + j\frac{2}{d^2}} \tag{6.1}$$

for $m = 1, 3, 5, \ldots\ldots, \infty$.

The magnetic field along the diagonal, $y = \ell - x$, found from Eq.(6) is:

$$H_{1z}|_{y=\ell-x} = 0 \tag{7}$$

Therefore, in view of equations (3.1) - (3.3) and (4), the partial field component H_{2z}, must satisfy the following boundary conditions:

$$H_{2z}|_{x=0} = H_{2z}|_{y=0} = 0 \tag{8.1}$$

and

$$H_{2z}|_{y=\ell-x} = K_0 . e^{j\omega t} \tag{8.2}$$

Solution of eddy current equation for H_{2z}, satisfying these boundary conditions, is as follows:

$$H_{2z} = -\sum_{m-odd}^{\infty} K_0 . \frac{4}{m\pi} . \left[\cos\frac{m\pi}{2\ell}(x+y) . \sin\left(\frac{m\pi}{2}\right) . \frac{\cosh \alpha'_m (x-y)/\sqrt{2}}{\cosh(\alpha'_m . \ell/\sqrt{2})} - \cos\frac{m\pi}{2\ell}(x-y) . \sin\left(\frac{m\pi}{2}\right) . \frac{\cosh\alpha'_m (x+y)/\sqrt{2}}{\cosh(\alpha'_m . \ell/\sqrt{2})}\right] . e^{j\omega t} \tag{9}$$

Where,

$$\alpha'_m = \sqrt{\left(\frac{m\pi}{\ell\sqrt{2}}\right)^2 + j\frac{2}{d^2}} \tag{9.1}$$

The distribution of magnetic field in the triangular core obtained using Equations (4), (6) and (9) is given below:

$$H_z = -\sum_{m-odd}^{\infty} K_0 . \frac{4}{m\pi} . \left[\sin\left(\frac{m\pi}{\ell} . y\right) . \frac{\sinh \alpha_m . (x-\ell/2)}{\sinh(\alpha_m . \ell/2)} + \sin\left(\frac{m\pi}{\ell} . x\right) . \frac{\sinh \alpha_m . (y-\ell/2)}{\sinh(\alpha_m . \ell/2)} + \sin\left(\frac{m\pi}{2}\right) . \left\{\cos\frac{m\pi}{2\ell}(x+y) . \frac{\cosh \alpha'_m (x-y)/\sqrt{2}}{\cosh(\alpha'_m . \ell/\sqrt{2})} - \cos\frac{m\pi}{2\ell}(x-y) . \frac{\cosh\alpha'_m (x+y)/\sqrt{2}}{\cosh(\alpha'_m . \ell/\sqrt{2})}\right\}\right] . e^{j\omega t} \tag{10}$$

χ **Eddy current distribution**

Now, the expressions for the components of eddy current density in the triangular core are found, in view of equations (2) and (10) as:

$$J_x = -\sum_{m-odd}^{\infty} K_0 . \frac{4}{m\pi} . \left[\frac{m\pi}{\ell} . \cos\left(\frac{m\pi}{\ell} . y\right) . \frac{\sinh \alpha_m . (x-\ell/2)}{\sinh(\alpha_m . \ell/2)} + \alpha_m . \sin\left(\frac{m\pi}{\ell} . x\right) . \frac{\cosh \alpha_m . (y-\ell/2)}{\sinh(\alpha_m . \ell/2)}\right.$$

$$-\frac{m\pi}{2\ell}.\sin\left(\frac{m\pi}{2}\right).\left\{\sin\frac{m\pi}{2\ell}(x+y).\frac{\cosh\alpha'_m(x-y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}+\sin\frac{m\pi}{2\ell}(x-y).\frac{\cosh\alpha'_m(x+.y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}\right\}$$

$$-\frac{\alpha'_m}{\sqrt{2}}.\sin\left(\frac{m\pi}{2}\right).\left\{\cos\frac{m\pi}{2\ell}(x+y).\frac{\sinh\alpha'_m(x-y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}+\cos\frac{m\pi}{2\ell}(x-y).\frac{\sinh\alpha'_m(x+.y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}\right\}\Bigg].e^{j\omega t}$$

(11.1)

$$J_y=\sum_{m-odd}^{\infty}K_0.\frac{4}{m\pi}.\left[\alpha_m.\sin\left(\frac{m\pi}{\ell}.y\right).\frac{\cosh\alpha_m.(x-\ell/2)}{\sinh(\alpha_m.\ell/2)}+\frac{m\pi}{\ell}.\cos\left(\frac{m\pi}{\ell}.x\right).\frac{\sinh\alpha_m.(y-\ell/2)}{\sinh(\alpha_m.\ell/2)}\right.$$

$$-\frac{m\pi}{2\ell}.\sin\left(\frac{m\pi}{2}\right).\left\{\sin\frac{m\pi}{2\ell}(x+y).\frac{\cosh\alpha'_m(x-y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}-\sin\frac{m\pi}{2\ell}(x-y).\frac{\cosh\alpha'_m(x+.y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}\right\}$$

$$+\frac{\alpha'_m}{\sqrt{2}}.\sin\left(\frac{m\pi}{2}\right).\left\{\cos\frac{m\pi}{2\ell}(x+y).\frac{\sinh\alpha'_m(x-y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}-\cos\frac{m\pi}{2\ell}(x-y).\frac{\sinh\alpha'_m(x+.y)/\sqrt{2}}{\cosh(\alpha'_m.\ell/\sqrt{2})}\right\}\Bigg].e^{j\omega t}$$

(11.2)

δ Conclusions

Field distribution in cores with triangular cross-sections is invariably found using numerical methods. This is because analytical solution for a general triangular core-section is not available. However, a right-angled isosceles triangle is an exception. Fourier series based analytical expressions are obtained for components of magnetic field intensity and eddy current density in long solid conducting cores with cross-sections in the shape of a right-angled isosceles triangle. For general triangular core-sections, equation (10) can be considered as an approximate initial solution which can be improved upon by numerical methods, if needed.

References

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